Continuity - Removable

1) a) $f(x)$ is not continuous at $x=0$ & $x=1$.

b) $f(x)$ has a removable discontinuity at $x=0$ (hole @ $(0,0)$)
$f(x)$ has a jump discontinuity at $x=1$

$\lim_{x\to 0} f(x) = 0$ $\quad$ $\lim_{x\to 1} f(x)$ DNE

c) $f(x)$ is continuous on $[-1,0)\cup(0,1)\cup(1,2]$

2) a) $f(x)$ is not continuous at $x=1$ & $x=2$.

b) $\lim_{x\to 1} f(x)$ DNE - $f(x)$ has a jump discontinuity at $x=1$

$\lim_{x\to 2} f(x) = 1$ - $f(x)$ has a removable discontinuity at $x=2$. (hole @ $(2,1)$)

c) $f(x)$ is continuous on $[-1,1)\cup(1,2)\cup(2,3]$

3) $f(x) = \dfrac{x-2}{x^2-5x+6} = \dfrac{x-2}{(x-3)(x-2)}$

a) $f(x)$ has discontinuties @ $x=3$ & $x=2$.

b) $\lim_{x\to 2} f(x) = \frac{1}{-1} = -1$ → removable (hole @ $(2,-1)$)

$\lim_{x\to 3} f(x)$ DNE → infinite discontinuity @ $x=3$ (vertical asymptote $x=3$)

c) $f(x)$ is continuous on $(-\infty,2)\cup(2,3)\cup(3,\infty)$

4) $f(x) = \dfrac{x^2-4}{x+2} = \dfrac{(x+2)(x-2)}{x+2}$

a) $f(x)$ has discontinuity @ $x=-2$

b) $\lim_{x\to -2} f(x) = -4$ → removable (hole @ $(-2,-4)$)

c) $f(x)$ is continuous on $(-\infty,-2)\cup(-2,\infty)$

5) $f(x) = \frac{x^2+x-12}{x^2+6x+8} = \frac{(x+4)(x-3)}{(x+4)(x+2)}$

a) $f(x)$ has discontinuities @ $x=-4$ & $x=-2$

b) $\lim_{x\to -4} \frac{(x-3)}{x+2} = \frac{7}{2}$ → removable (hole @ $(-4, \frac{7}{2})$)

$\lim_{x\to -2} f(x)$ DNE → infinite (vertical asymptote: $x=-2$)

c) $f(x)$ is continuous on $(-\infty,-4) \cup (-4,-2) \cup (-2,\infty)$

6) $f(x) = \begin{cases} 2x^2-4, & x<2 \\ ax+3, & x\geq 2 \end{cases}$

$f(2) = 2a+3$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

$4 = 2a+3$

$1 = 2a$

$\boxed{a = \frac{1}{2}}$

7) $f(x) = \begin{cases} ax^2+2, & x<3 \\ 4x-1, & x\geq 3 \end{cases}$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$

$9a+2 = 11$

$9a = 9$

$\boxed{a = 1}$

8) $f(x) = \begin{cases} \frac{(2x+1)(x-2)}{x-2}, & x\neq 2 \\ k, & x=2 \end{cases}$

I. $f(2) = k$ II. $\lim_{x\to 2} (2x+1) = 5$

$\boxed{k = 5}$ ✓

9) $\lim_{x\to\infty} \frac{x^{99}}{e^x} = 0$

10) $\lim_{x\to\infty} \frac{e^x}{\ln x} = \infty$

11) $\lim_{x\to\infty} \frac{\ln x}{x^{99}} = 0$

12) $\lim_{x\to -\infty} \frac{x^{99}}{e^x} = -\infty$